Validation of Simulation Based Models: A Theoretical Outlook

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Abstract: Validation is the most incomprehensible part of developing a model. Nevetheless, no model can be accepted unless it has passed the tests of validation, since the procedure of validation is vital to ascertain the credibility of the model. Validation procedures are usually framework based and dynamic, but a methodical procedure can be followed by a modeller (researcher) in order to authenticate the model. The paper starts with a discussion on the views and burning issues by various researchers on model validation and the foundational terminology involved. The paper later highlights on the methodology and the process of validation adopted. Reasons for the failure of the model have also been explored. The paper finally focuses on the widely approved validation schemes (both quantitative and qualitative) and techniques in practice, since no one test can determine the credibility and validity of a simulation model. Moreover, as the model passes more tests (both quantitative and qualitative) the confidence in the model increases correspondingly.

Keywords: Validation, simulation, dynamic models, validation schemes, validation process, modelling.

1. Introduction

Validation has been one of the unresolved problems of systems modelling (Mohapatra 1987). It is true for simulation models in general and system dynamic models in particular. System dynamic modelling makes use of computer simulation (packages like Matab, Stella) to generate the consequences for studying the dynamic behaviour of the system. In contrast, validations of Optimisation Models, Decision Theory or Game Theory are often not questioned since the solution procedures are elegant and correct. Reasons for conceptual and simulation models having received more criticism could be the ease with which the models and their overall results being understandable. Another reason being, the simulation model of any system could only be an approximation of the actual system, no matter the amount of time spent on the model building. Hence if the model produced is not a 'close' enough approximation to this actual system, conclusions derived from such model are likely to be divergent and erroneous, leading to possible costly decision mistakes been made (Ijeoma et al. 2001).

According to Law (2001), validation can be done for all simulation models regardless of whether their corresponding systems exist presently or would be built in future. Also, Kleijnen (1999) and Sterman (1984) give insight on validation of simulation models using statistical techniques and reasoned that the technique applied would depend on the availability of data in the real system. Contradicting the above authors, some authors have also stated that "there is no such thing as an absolutely valid model, credibility of a model can be claimed only for the intended use of the model or simulation and for the prescribed conditions under which the model or simulation has been tested" (DMSO 1996). Sterman (2000) also argue "validation and verification are impossible; the emphasis should be more on model testing i.e. the process to build confidence that a model is appropriate for the purpose. Some models may be better than others; some models, while not completely valid, possess a greater degree of authenticity than others. Furthermore, all models are, in a sense, wrong because there could always be a counter test to which the model did not conform to completely".

Nevertheless, the power of a model or modelling technique is a function of validity, credibility, and generality (Solberg 1992). Hence model validation is not an option but a necessity in a dynamic modelling scenario. Usually the simplest model, which expresses a valid relation, will be the most powerful; however, there is no single test that would allow the modellers to assert that their models have been validated. Rather, the level of confidence in the model can increase gradually as the model passes more tests (Forrester, and Senge 1980). The relationships of cost (a similar relationship holds for the amount of time) of performing model validation and the value of a model to the user as a function of model confidence are shown in Figure1. As shown in the figure, the value of the model increases as the level of confidence in the model is increased, correspondingly the cost of model validation also increases.

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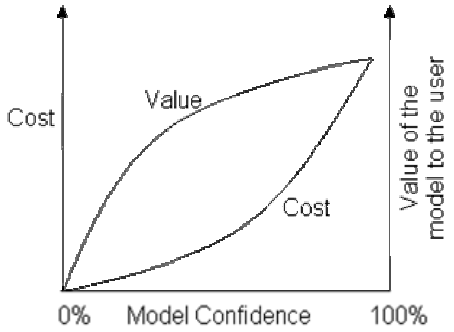

Figure 1: Value, cost vs. model confidence (Source: Sargent 2003)

Validation cannot be carried out by the modeller (or researcher) alone, communication with the client (or user) plays a large role in building a valid model and establishing its credibility (Carson 1989). Another relevant issue of concern is that by how much the model output could deviate from system output and still remain valid (Kleindorfer et al. 1998). Since the model created is an approximation of the actual system, some errors and approximations are unavoidable. Model validation thus resides in decision between the modeller and client; when both groups are satisfied, the model is considered valid (Goldberg et al. 1990).

A wide range of tests to build confidence in a model have been developed by authors like Forrester and Senge (1980), Barlas (1989 and1996), Khazanchi (1996) and Saysel et al. (2004) a summary of which is presented under Validation Schemes.

2. Validation defined

The definitions of validation as stated by different authors are listed below:

- Substantiation that a computerised model within its domain of applicability possesses a satisfactory range of accuracy consistent with the intended application of the model (Sargent 2003).
- Validation is the process of determining that the model on which the simulation is based is an acceptably accurate representation of reality (Giannanasi et al. 2001).
- Validation is the process of establishing confidence in the usefulness of a model (Coyle 1977).
- The process of determining the degree to which a model is an accurate representation of the real-world from the perspective of the intended uses of the model (DoD 2002).

3. Viewpoints on validation

The viewpoints on validation are based on modified views of traditional validation techniques. These characteristics of validation are listed accordingly below:

- A model should be judged for its usefulness rather than its absolute validity.
- A model cannot have absolute validity but it should be valid for the purpose for which it is constructed.
- There can be no one test with which the model validity can be judged.
- As a model passes the various tests, confidence in the model is enhanced.
- "Failing a test helps to reject a wrong hypothesis, but passing is no guarantee that the model is valid" (Sushil 1993).
- "Quantitative as well as qualitative validity criterion should be given more credence (Forrester 1961)".
- Most of the information from the real system is used to check the consistency of model behaviour.
- Rejecting a model because it fails to reproduce an exact replica of past data is not acceptable.
- Rejecting a model because it fails to predict a specific future event is not acceptable because social systems operate in wide noise frequencies.

4. Methodology for validation

Validation deals with the assessment of the comparison between 'sufficiently accurate' computational results from the simulation and the actual/ hypothetical data from the system. Validation does not specifically address how the simulation model can be changed to improve the agreement between the computational results and the actual data. The fundamental strategy of validation involves identification and quantification of the error and uncertainty in the conceptual/ simulation models, quantification of the numerical error in the computational solution, estimation of the simulation uncertainty, and finally, comparison between the computational results and the actual data. Thus, accuracy is measured in relation to actual/ hypothetical data, our best measure of reality. This strategy does not assume that the actual/ hypothetical data are more accurate than the computational results. The strategy only asserts that simulation results are the most faithful reflections of reality for the purposes of validation (AIAA 1998).

5. Model validation process

Figure 2 shows the model validation process in a simpler form. The 'problem entity' is the system (real or proposed – e.g. dynamics of integrated Knowledge Management and Human Resource Management can be considered as a problem entity (Martis 2004)) to be modelled; the 'conceptual model' is the mathematical/ verbal representation (influence diagram) of the problem entity developed for a particular study; and the 'computerised model' is the conceptual model implemented on a computer (simulation model). The inferences about the problem entity are obtained by conducting simulations on the computerised model in the experimentation phase.

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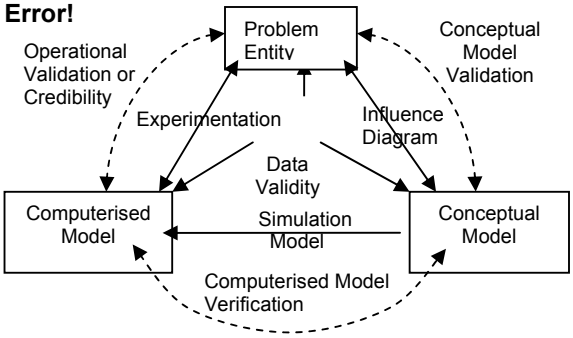

Figure 2: Model validation process.

There are three steps in deciding if a simulation is an accurate representation of the actual system considered, namely, verification, validation and credibility (Garzia et al. 1990). 'Conceptual model validation' is the process of determining that the theories and assumptions underlying the conceptual model are correct and that the model representation of the problem entity is "reasonable" for the intended purpose of the model. 'Computerised model verification' is the process of determining that the model implementation accurately represents the developers' conceptual description of the model and the solution to the model (AIAA 1998). 'Operational validation' is defined as determining that the model's output behaviour has sufficient accuracy for the model's intended purpose over the domain of the model's intended applicability (Sargent 2003). Operational validity determines the models credibility. 'Data validity' is defined as ensuring that the data necessary for model building, model evaluation and conducting the model experiments to solve the problem are adequate and correct (Love et al. 2000).

6. Reasons for failure of models

Some of the reasons due to which the models fail the validation tests are enumerated below as follows:

- Model-structure- In both the conceptual model and the simulation model mathematical simplifications might be inadequate for capturing complex dynamics.
- Numerical solution- The solution of the simulation model might differ dramatically from the ideal solution.
- Input values- Proper numerical values of the inputs that describe the scenario for prediction might be known only approximately.
- Observation errors- Inaccurate observations of real system.
- System noise- Failure to recognize random changes existent in the system.
- Project management errors- These errors revolve around project management and related communication issues (Carson 2002).
- Inappropriate simulation software – either too inflexible or too difficult to use (Law 2003).
- Misinterpretation of simulation results

7. Validation schemes

7.1 Validation scheme as proposed by Forrester and Senge (1980):

This validation criterion is used to validate quantitative as well as qualitative models. The validation scheme is mainly divided into four phases; as the model passes more tests under every phase the confidence in the model increases correspondingly. The validation scheme as proposed by Forrester and Senge (1980) is enumerated below:

7.1.1 Importance of model objective:

The validity of a model cannot be greater than the objective set for it. Therefore, the model objective must be a justified representation of the values prevalent in the real system. The method of setting model objectives by the conceptualization of problems in the existent system seems unstructured, unless the problem elicitation is done under the guidance of experts from the various subsystems existent within the system. A model could be proven valid by a series of methods, but the validation may be totally useless if the objectives are wrongly set.

7.1.2 Validating model structure:

These tests help in establishing confidence in the model structure.

Tests of suitability:
Structure-Verification Test: This test is meant to answer the following question "Is the model structure not in contradiction to the knowledge about the structure of the real system, and have the most relevant structures of the real system been modelled?"

Dimensional-Consistency Test: "Do the dimensions of the variables in every equation balance on each side of the equation?" This test verifies whether all equations are dimensionally constant.

Extreme-Conditions Test: "Does every equation in the model make sense even if subjected to extreme but possible values of variables?" Policy equations are scrutinized for their applicability in extreme conditions.

Boundary-Adequacy Test: This test verifies whether the model structure is appropriate for the model purpose (Barlas 1989). "Is the model aggregation appropriate and includes all relevant structure containing the variables and feedback effects necessary to address the problem and suit the purposes of the study?"

Tests of consistency:
Face validity test: "Does the model structure looks like the real system? Is it a recognisable representation of the real system? Does a reasonable fit exist between the feedback structure of the model and the essential characteristics of the real system?"

Parameter-Verification Test: Parameters and their numerical values should have real system equivalents. "Do the parameters correspond conceptually and numerically to real life? Are the parameters recognisable in term of real systems, or are some parameters contrived to balance the equations? If the values selected for the parameters consistent with the test information available about the real system?"

Test of utility and effectiveness:
Appropriateness for audience: "Is the size of the model, its simplicity or complexity, and its level of aggregation or richness of detail appropriate for the audience for the study?" The more the appropriate a model for the audience the more will be the audience's perception of model validity.

7.1.3 Validating model behaviour:

These tests help in establishing confidence in the model behaviour.

Tests of suitability:
Parameter sensitivity test: "Is the behaviour of the model sensitive to reasonable variations in parameter values, i.e. do the modes of the behaviour change with parameter variations?"

Structural sensitivity test: "Is the behaviour of the model sensitive to reasonable structural reformulation, i.e. do the modes of the behaviour change with structural variations?"

Tests of consistency:
Behaviour-Reproduction Test: Here the generated model behaviour is judged with the historical behaviour. "How well the model generated behaviour matches observed behaviour of the real system in terms of symptom generation, frequency generation, relative phasing, multiple mode, and behaviour characteristics?"

Behaviour-Prediction Test: This test calls for pattern prediction. "Whether or not a model generates qualitatively current patterns of future patterns of future behaviour in terms of periods, shape or other characteristics?

Behaviour-Anomaly Test: Behaviour conflicting with the real system helps in finding obvious flaws in the model. "Does behaviour shown by the model is conflicting with the real system behaviour and how implausible behaviour arises if the assumptions are altered?"

Family member test: Whenever possible, attempt should be made to build a general model of the class of system to which a particular member belongs. The general theory is depicted in the structure. Parameter values are chosen to depict a particular situation. By choosing a different set of parameter values the model can be applied to other situation as well.

Surprising behaviour test: "Does the model under some test circumstances produces dramatically unexpected or surprise behaviour, not observed in the real system? Whether such a surprise behaviour is due to model structure or some causes in the real system can be assigned to such a behaviour?"

Extreme-Policy Test: "Does the model behave in an expected fashion under extreme policies, even ones that have never been observed in the real system?" If the model behaves in an expected fashion under extreme policies, then it boosts confidence in the model (Saysel et al. 2004).

Boundary adequacy (behaviour) test: "Does the model include the structures necessary to address the issues for which it is designed?" If an extra model structure does not change the behaviour, then this extra structure is not necessary. Alternatively, if a model structure does not reproduce desired model behaviour, it calls for inclusion of additional model structure (Barlas 1996).

Behaviour-Sensitivity Test: "Whether plausible shifts in parameters can cause model to fail

behaviour tests previously passed?" Here the sensitivity of the model to changes in parameter values is judged '(Saysel et al. 2004)'.

Statistical tests: "Does the model behaviour statistically like data from real system?" (Law and Kelton 2000).

Tests of utility and effectiveness:
Counter intuitive behaviour: "In response to some policies, does the model exhibit behaviour that at first contradicts intuitions and later, with the aid of the model, is seen as a clear implication of the structure of the system?" (Richardson et al. 1981).

"Is the model capable of generating new insights or at least the feeling of new insights, about the nature of the problem addressed and the system within it arises?"

7.1.4 Validating policy implications:

Tests of suitability:
Policy sensitivity and robustness test: The sensitivity of a policy with respect to change in parameter values is judged during this test. "Whether the model based policy recommendations change with reasonable changes in parameter values or reasonable alteration in equation formulations?"

Tests of consistency:
Changed Behaviour Prediction Test: "Whether the model correctly predicts how behaviour of the system will change if a governing policy is changed?"

Boundary adequacy (policy) test: "Whether modifying the model boundary (i.e. conceptualisation of additional structure) would alter policy recommendations arrived by using the model?"

System Improvement Test: "Whether the policies found beneficial after working with a model, when implemented, also improve real system behaviour?"

Test of utility and effectiveness:
Implementable policy test: "Can those responsible for policy in the real system be convinced of the value of model-based policy recommendations? How is the real system likely to respond to the process of implementation?" the policy recommendations should be such formulated and argued so as to fit in the mental models of those to whom they are addressed.

7.2 Validation scheme as proposed by Khazanchi (1996):

This validation criterion is mainly used to validate qualitative/conceptual models and consists of a set of criteria for validation. The criteria for validation as suggested by Khazanchi (1996) are as follows:

1. Is it plausible/ reasonable? This criterion is useful to assess the apparent reasonableness of an idea and could be demonstrated by deduction from past research or theories.
2. Is it feasible? A feasible concept would be operational only if it would be open to graphical, mathematical, illustrative characterisation.
3. Is it effective? An effective conceptual model should have the potential of serving our scientific purposes (Kaplan 1964).
4. Is it pragmatic? This criterion emphasises that concepts and conceptual models should have some degree of logical self-consistency or coherence with other concepts and conceptual models in the discipline (Hunt. 1990).
5. Is it empirical? Empirical content implies that a concept or conceptual model must have "empirical testability" (Hunt 1990).
6. Is it predictive? A conceptual model that is predictive would, at the least, demonstrate that given certain antecedent conditions, the corresponding phenomenon was somehow expected to occur.
7. Is it inter-subjectively certifiable? This criterion states "Investigators with differing philosophical stance must be able to verify the imputed truth content of these concepts or conceptual structures through observation, logical evaluation, or experimentation (Hunt 1990).
8. Is it inter-methodologically certifiable? This criterion provides that investigators using different research methodologies must be able to test the veracity of the concept or conceptual model and predict the occurrence of the same phenomenon.

8. Other validation techniques

Combinations of these techniques are generally used for validating a simulation model. These tests can be used in addition to the validation schemes in the preceding section to increase the credibility of the model.

1. Comparison to other models: Different outputs of the simulation model being validated are compared to those of other 'valid' models.

2. Degenerate test: This has to do with appropriately selecting values of the input and internal parameters to test the degeneracy of the model's behaviour. For instance, test to see if the average number in the queue of a single server continues to increase with respect to time when the arriving rate is larger than the service rate (Ijeoma et al. 2001).
3. Events validity: The events of occurrences of the simulation model are compared to those of the real system to see if they are similar e.g. verify the exit rate of employees.
4. Face validity: This has to do with asking knowledgeable people if the system model behaviour is reasonable (Forrester 1961).
5. Historical Data validation: The experimental data is compared with the historical data; to check whether if the model behaves in the same way the system does (Balci et al. 1982).
6. Predictive validation: The model is used to predict the system's behaviour, and then comparison is made between the real system behaviour and the model's forecast to determine if they are the same (Sargent 2003).
7. Schellenberger's Criteria: This include technical validation which has to do with identifying all divergences between the model assumptions and perceived reality as well as the validity of the data used, operational validity which addresses the question of how important these divergences are and dynamic validation which ensures that the model will continue being valid during its lifetime (Ijeoma et al. 2001).
8. Scoring Model Approach: Scores (or weights) are determined subjectively when conducting various aspects of the validation process and then combined to determine category scores and an overall score for the simulation model. A simulation model is considered valid if its overall and category scores are greater than some passing score(s) (Gass 1993).
9. Clarity: Clarity refers to the extent to which the model clearly communicates the implied causality/linkages.
10. Black-box validation: This test is concerned with determining whether if the entire model is an adequately accurate representation of the real world (Ijeoma et al. 2001).
11. Extreme Condition Test: The model structure and output should be reasonable for any extreme and unlikely combination of values in the system. For example, if in-process inventories are zero, production output should be zero (Sargent 2003).

9. Conclusions

As rightly coined by (DMSO 1996), validation is both an art and a science, requiring creativity and insight. But validation is a convoluted, multifarious and exasperating procedure, and is unavoidable as it is the evidence for the steadfastness and legitimacy of the model. Moreover, no single procedure can suit all the models. Statistical based validation techniques have been widely accepted among the management community. But the problem associated with this method is being able to determine the suitable type of statistical procedure, which in turn depends on the right type of data that is available for analysis. Moreover, the amount of deviation from the real system that is within the acceptable limits is uncertain.

The paper has given an insight on the widely approved validation schemes and techniques in practice. The validation schemes can be applicable to quantitative (mathematical/ computerised) as well as qualitative (conceptual) models. But reliability of the model can only be ascertained as the model passes more and more tests. Also, the decision of accepting a model as valid cannot be left to the modeller alone, inclusion of the client / practitioners in the validation procedure should be ascertained. Researchers and practitioners may find this paper quite useful as the procedures for validation discussed are quite generic, and hence, may be applied to other dynamic models as well.

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