GeoGebra Materials for LMS Moodle Focused Monge on Projection

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Abstract: "The use of e-learning and modern technologies is an integral part of teaching descriptive geometry" (Rankowski and Minaruth, 1979). Teachers often create Flash animations, websites and other interactive materials for their students using special tools and software; e.g. Cabri, GeoGebra or Construct 3D (García et al, 2007). There is an option of installing GeoGebra plugin within the frame of LMS Moodle, which results in having two important parts of self-study in one place. Thus, it might be a solid background for creation of integrated materials, which should support the development of spatial imagination. However, "this is the ability most students lack, which results in having problems with understanding simple tasks in descriptive geometry" (Gittler and Judith, 1998), which also leads to the fact, that Monge projection is not a favourite topic for them. This article continues the paper (Ferdiánová and Poruba, 2016), in which paper models of basic tasks in Monge projection, designed as supportive material for teaching geometry, were presented. The aim of this paper is to introduce interactive materials for Monge projection, which are implemented into LMS Moodle, as well as show results of pilot research of influence of using these materials on students. Students have geometric tasks, anaglyph models and templates for creation of paper models, which were used for better visualisation of geometric relationships in space, at disposal. Outcomes of pilot research questionnaires and tests are processed by basic statistical methods, e.g. Wilcox test. Outcomes of pilot testing influence further changes of usage interactive materials and LMS Moodle in self-study process.

1. Introduction

Imagination is amazing and unique human ability, which is connected to creation of images, ideas and concepts. Molnár (2009) defines geometrical imagination as a set of abilities which relate to reproduction and anticipation, static and dynamic ideas about shapes, attributes and relations among geometrical figures in space. Slezáková (2011) argues that students have relatively underdeveloped geometric imagination. They understand some terms only formally and have disproportionate problems with solving geometric problems. This problem has many reasons. One of them is very little care we give to teaching geometry and geometric expression at all. On the other hand, there is lack of motivation of students as well as their dislike to this topic. "Students have different dispositions to develop spatial and geometry imagination, the rate of development, however, depends on education and learning. We know three dimensional space very well, because this space is natural for us and it is about us. The ability to visualize spatial situations is not innate as given. It must be deliberately developed" (Ferdiánová and Poruba, 2016). One way of improving the imagination is the use of geometric models in teaching.

Jančařík (2016) uses dynamic models using 3D projection in courses of teacher's training preparation. Models are very often used in teaching Descriptive Geometry in the Architecture courses. "Descriptive Geometry for architects should provide basic information on the geometry of shapes as well as on projective methods" (Schmid-Kirsch, 1997). Schmid-Kirsch (1997) created models for Helical line on a cylinder, Clockwise and anti-clockwise corkscrew and so on for teaching DG at the Faculty of Architecture in Hannover University. He wanted to motivate students on lectures, when each topic was presented in different ways, so that all senses are involved, not only the brain. Schmid-Kirsch (1997) asserted of a conclusion, that the knowledge of various methods (traditional and new) gives the chance to select the most suitable one. Instead of presenting a large number of finished solutions, it is recommendable to reduce the number of treated problems and solve them step by step. This produces more independence, improves the sense of orientation and the problem solving strategies. Brahkage (2004) presented using WinCAG (Windows version of Computer Aided Geometry) in teaching Descriptive Geometry. Option tools and view aspects of WinCAG, which teacher can use in a lecture, were shown in the article. This application can contribute to didactical aid and to geometric spatial cognition. Vogt (2015) created a special e-learning course for students of Geometry and Engineering Graphics in AutoCAD. The e-learning course obtains a book of theory, tutorials, drafts for each of the covered topics, advantage of animations, instructional films and online quizzes. Pütz (2001) described using of didactic principles and shows effective methods demonstrated on examples of Monge projection in lecture Descritpive geometry for architects. The lecture was attended by about 250 students - he need use another teaching methods than me. He used excellent motivational example of a „tree house", which consists of a cuboid standing on one of its corners. Students constructed drawing in Monge projection and after that they built real paper model in teamwork.

All these approaches seem to be very enriching. Unfortunately, they have not been statistically tested yet - it cannot be statistically confirmed, which of these methods is more effective or brigs more benefits.

The excellent book for teaching Monge projection, in which 3D models of tasks are included, is Basic examples of descriptive geometry in models (in original Základní úlohy deskriptivní geometrie v modelech by Kupčáková, 2002). Geometric shapes in Monge projection by Jakub Poruba is very similar bachelor thesis like the book Basic examples of descriptive geometry in models. We represented a few parts from this thesis on conference Aplimat 2016. The work contains 12 paper models of Monge projection with complete drawings and so on. Our aim is to introduce interactive models for Monge projection, which are very easy to use in e-learning, as well as to show results from practical lesson in which they were used. Models of simple tasks were created in dynamic program GeoGebra, anaglyphs were used for better visualisation of geometric relationships in space.

2.   Anaglyph

Anaglyphic stereograms (anaglyphs) are stereo pairs of images in which each image is shown using a different colour. This method consists of printing two negatives which form the stereoscopic photograph in the same paper, one in blue (or green), one in red. Two-colour negative are created from two central projection images projected from the centres approximately 6.5 centimetres apart, which is the centre distance typically between human eyes. The viewer would then use coloured glasses with red (for the left eye) and blue or green (right eye). The left eye would see the blue image which would appear black, whilst it would not see the red; similarly the right eye would see the red image, this registering as black. Our brain then creates a three dimensional image and we would see the illusion in front of or in the paper, see Figure 1. This is an illusion in principle.
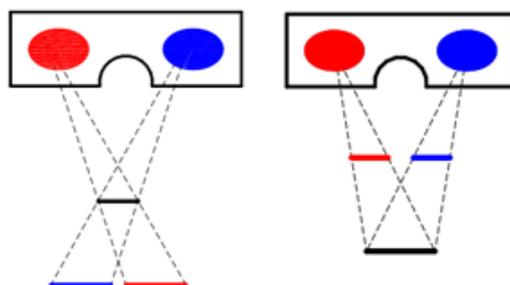

Figure 1: Orthoscopic (left) and pseudoscopic view (right)

Wilhelm Rollmann first illustrated the principle of the anaglyph using blue and red lines on a black field with red and blue glasses to perceive the effect in 1853, but this method was for line drawings only. In 1858, Joseph D'Almeida came upon a new way to view space based on the same principles. In this system, two images are created using two different lights, red and green/blue. By looking through the light filters, one achieves the 3D effect. The technical term for this technique was anaglyph. Alexandr (1999) argues that the word anaglyph comes from the Greek anagluphos, meaning "wrought in low relief"; this comes from the word anagluphein, which means "to carve in relief" (ana = up + gluphein = to carve). Wiliam Friese-Green patented a camera/projector almost identical to Varley's stereoscopic model in 1893. He demonstrated film X-Rays and the first film was shown in Britain. Anaglyph films were very popular in the 1920s. Anaglyphic stereograms were used in education and science. For example the US Geological Survey has thousands 3D full-color images, which depict the geology and scenic features of the U.S. National Park system. Scientific images also include the presentation of complex multi-dimensional data sets and stereographic images of the surface of Mars by NASA. During the time, graphic designers have created anaglyph comics, movies, videogames and so on.

3.   GeoGebra

"*GeoGebra is free and multi-platform dynamic mathematics software for all levels of education. It also is a rapidly expanding community of millions of users located in just about every country. GeoGebra has become the leading provider of dynamic mathematical software, supporting science, technology, engineering and mathematics (STEM) education and innovations in teaching and learning worldwide.*" (GeoGebra, 2016) This software contains special areas for Geometry, Algebra and Spreadsheet, all of them are connected and fully dynamic. Interface of GeoGebra is user-friendly, you can use too many tools for construction. Main advantage of this software is in visualization of geometrical problems. Every construction or sheet can be saved as file

with file name extension .ggb, exported to Dynamic Worksheet or can be uploaded to GeoGebra tube. Every user can create an account in GeoGebra tube website and he can also see materials from other GeoGebra users there. It is a big benefit for authors, because if they lose all their GeoGebra files in their computers, they still can find them safely saved on GeoGebra Tube. The tube allows creating examples, sheets or special tools. Important part of GeoGebra is option of creating Custom Tools. Any tool can by created on the basis of already created general construction. Such tool is then represented by its own icon in main menu, which makes the tool handy. Another advantage is, that we also create a direct command by this way, which can be used in Command line. Created constructions can be shared on GeoGebra Tube or saved in form of .ggb file. Similar constructions are usually used in constructive geometry for general constructions such as inscribed or circumscribed circle of a triangle, etc. For Monge projection, tools for projection of a point, a straight line and a plane were created. Detailed description of creation of these tools can be found in the paper New Tools for Monge projection in GeoGebra (Ferdiánová and Poruba, 2017). Students thus have many already created tools at disposal in LMS Moodle, which are, however, accessed after they fully know basics of Monge projection.

GeoGebra contains a special plugin for LMS Moodle, which enables to use it in three ways: Assignment activity, Geogebra Tube Repository and GeoGebra questiontype. Its advantage is that it is not necessary to install GeoGebra on own PC. In beta testing of this plugin, all materials standardly sent to students were used. Another advantage is creation of automatic questionnaires, which can be used for testing the level of gained knowledge. Interconnection of LMS Moodle and GeoGebra was ideal step, which will motivate students more for self-study.

GeoGebra was created by Markus Hohenwarter in 2001/2002 as part of his master's thesis in mathematics education and computer science at the University of Salzburg in Austria. Supported by a DOC scholarship from the Austrian Academy of Sciences he was able to continue the development of the software as part of his PhD project in mathematics education. (Hohenwater and Preiner, 2007) Markus Hohenwarter then continued it at Florida Atlantic University (2006–2008), Florida State University (2008–2009). On the present, he works at the University of Linz, together with the help of open-source developers and translators all over the world. In October 2011, GeoGebraTube, a new part of the website, was introduced to the public. Since the start, more than 15000 materials uploaded on GeoGebraTube exists. In June 2012, they created a Chrome a Chrome Application with extra collaborative features, such as saving/storing and sharing .ggb files via Google Drive. GeoGebra set up a project on crowdfunding campaign on Kickstarter, which was focused on developing a new version for tablets. It was a huge success – 310 backers donated 12 010 dollars (see Figure 2). The tablet version is fully functional, which was especially welcomed by schools in the Czech Republic, which own tablets from project Tablets for schools.
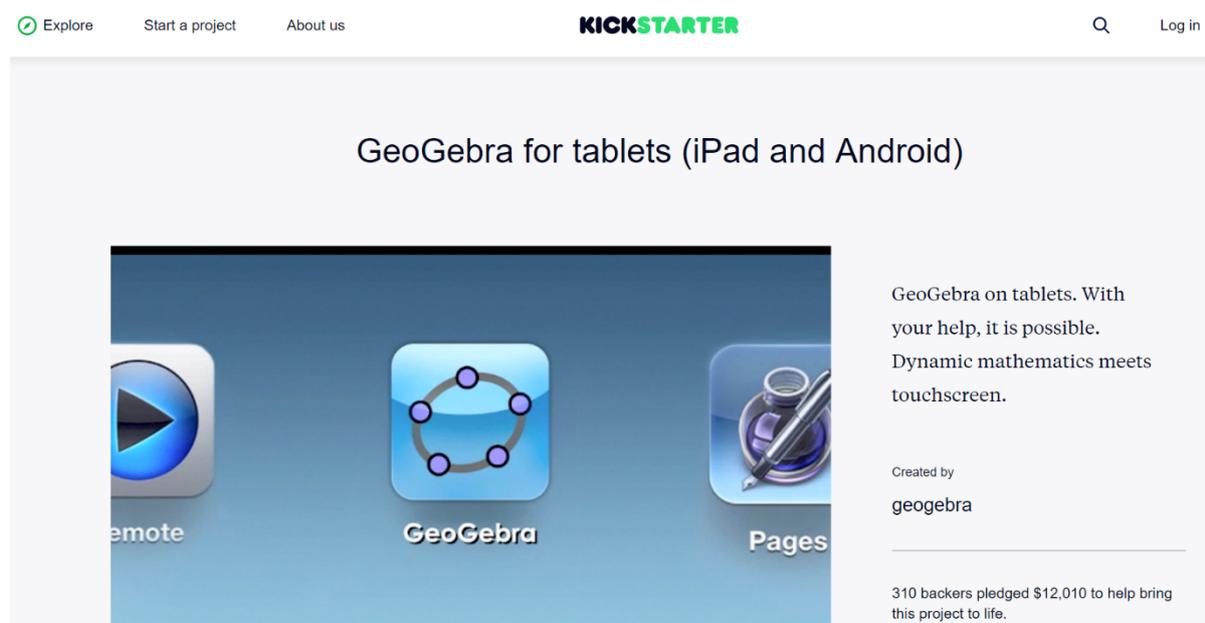

Figure 2: Printscreen of project GeoGebra from website Kickstarter

GeoGebra has received several educational software awards in Europe and the USA (GeoGebra, 2016):
- Archimedes 2016: MNU Award in category Mathematics (Hamburg, Germany)
- Microsoft Partner of the Year Award 2015: Finalist, Public Sector: Education (Redmond, WA, USA)
- MERLOT Classics Award 2013: Multimedia Educational Resource for Learning and Online Teaching (Las Vegas, Nevada, USA)
- NTLC Award 2010: National Technology Leadership Award (Washington D.C., USA)
- Tech Award 2009: Laureat in the Education Category (San Jose, California, USA)
- BETT Award 2009: Finalist in London for British Educational Technology Award
- SourceForge.net Community Choice Awards 2008: Finalist, Best Project for Educators
- AECT Distinguished Development Award 2008: Association for Educational Communications and Technology (Orlando, USA)
- Learnie Award 2006: Austrian Educational Software Award (Vienna, Austria)
- eTwinning Award 2006: 1st prize for "Crop Circles Challenge" with GeoGebra (Linz, Austria)
- Trophées du Libre 2005: International Free Software Award, category Education (Soisson, France)
- Comenius 2004: German Educational Media Award (Berlin, Germany)
- Learnie Award 2005: Austrian Educational Software Award (Vienna, Austria)
- digita 2004: German Educational Software Award (Cologne, Germany)
- Learnie Award 2003: Austrian Educational Software Award (Vienna, Austria)
- EASA 2002: European Academic Software Award (Ronneby, Sweden)

4.   Special materials for Monge projection

We have created a collection of examples for Monge projection. Each example contains task, description of the construction, drawing, paper model and anaglyph version. Paper models and anaglyph version were created in application GeoGebra. Assembling the model does not take more than 10 minutes and all what students need arescissors, a ruler and a glue. If we look at the model at eye level, it should be easier to understand the constructions in the drawing. Anaglyph version of model is save in file of GeoGebra or in LMS Moodle for viewing, student can open the file and they can move with model. It is still popular – because it is easy to make an anaglyph visualization setup. From the special equipment you need only red/blue glasses. We have saved all materials for our students in LMS Moodle. LMS Moodle is important part of teaching Monge projection, because, thanks to the new GeoGebra plugin, students have construction tests at disposal, which they can use in self-study process. Although it misses explanation of possible mistakes in construction, the plugin includes a mode, which identifies, if the construction is right or wrong. However, it is the teacher who must give a feedback. (Miarka and Žáček, 2015) Before implementing the plugin into the practice, students only uploaded their .ggb files into LMS Moodle and the teacher had to download them, check them and then give a feedback in which step a mistake was done.
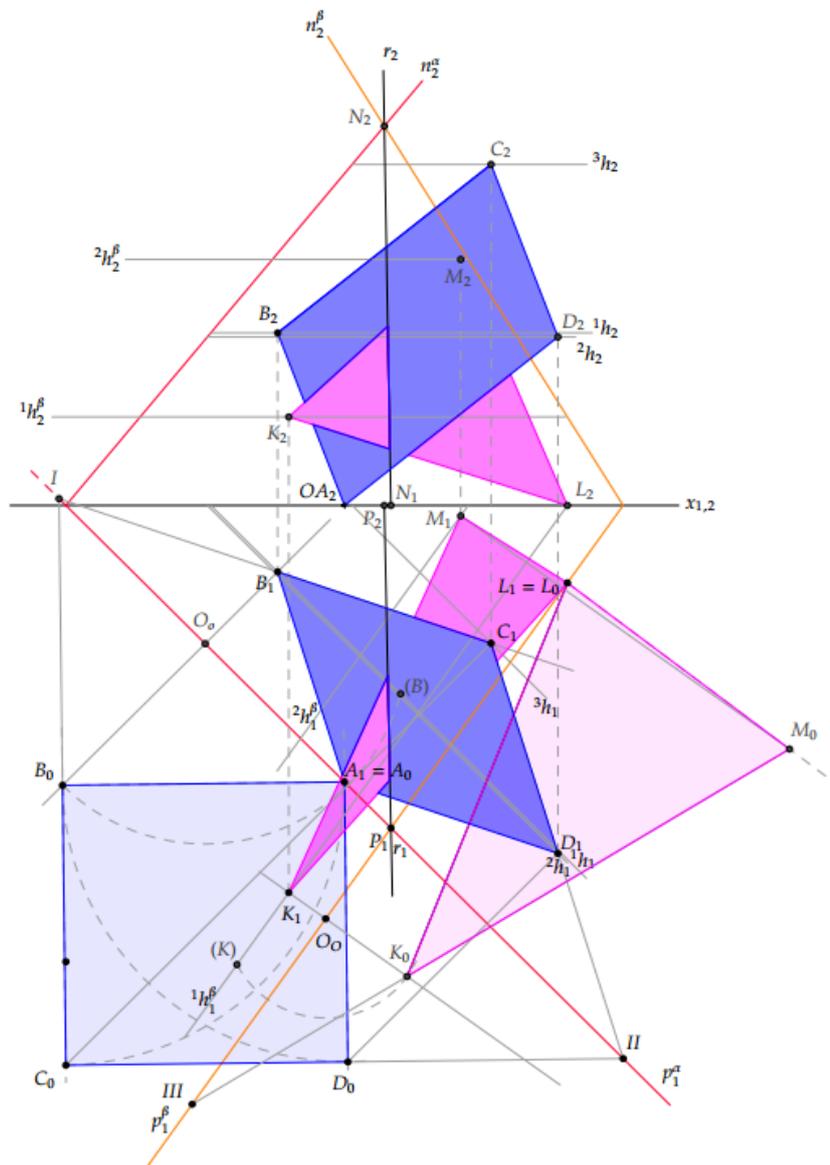
Figure 3: The drawing of the task Intersection of a triangle and a square created by Jakub Poruba, 2016

The following example represents set of models that have been created so far.

Determine the mutual position of a square and a triangle, if the square is given by points *A*[0; 5; ?], *B*[-1,2; 1, 2; ?] in plane *α*(-5; 5; 6) and the triangle is in plane *β*(5; 7; 8) given by points *K*[-1; 7; ?]; *L*[4; 1, 4; ?] , angle *KLM* = 75∘ and the segment line *KM* of 8 cm length. (Poruba, 2016)

The material contains detailed description of the construction for a drawing. Students are able to construct a drawing (see Figure 3) using the description of the construction without the help of the teacher. If they have a poor geometric imagination, they can assemble the paper model (see Figure 4). The last option is to use the anaglyph version of example (see Figure 5).
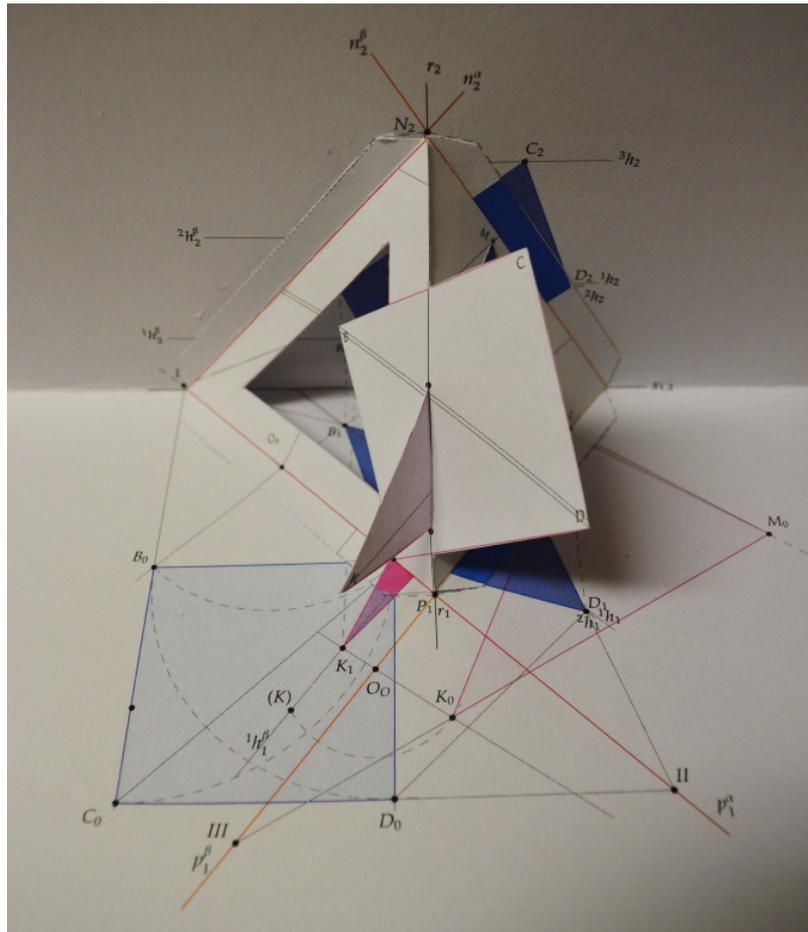
Figure 4: The paper model of the task Intersection of a square and a triangle. photo by Jakub Poruba
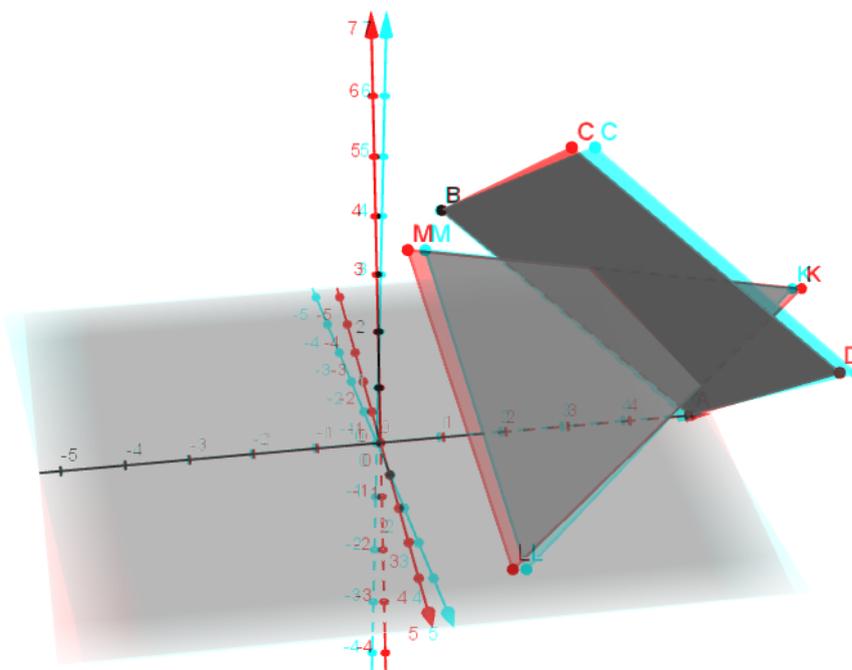

Figure 5: The anaglyph of the example

We used all materials in course on Constructional Geometry. Theoretical background and principles of working with these materials were introduced during the lesson and all needed materials were available for students during the particular lesson as well as after its end.

5. Pilot testing of materials

The Constructional Geometry course was introduced as an obligatory course for all bachelor degree programmes in combination with mathematics. The course contains these parts: Planimetric problems, properties of conics, fundamental stereometric problems, affinity and collineations, Affinity of a circle and ellipse, introduction to Monge projection, positional problems in Monge projection, Metric problems in Monge projection, solids in Monge projection. We can see Monge projection is 50 per cent of the course. In previous years, the success of solving problems in Monge projection of 50-60% for our students. We included new interactive materials for Monge projection to our course for academic year 2016/17.

A test and a questionnaire were created for students of Constructional Geometry. The test obtained three tasks on Monge projection with different levels of difficulty. The task 1 was focused on finding step of a plane, task 2 contained positional problems in Monge projection. Task 3 was focused on using affinity in construction; for example, student has to construct a square in general plane. The test was part of the standard test in summer terms 2015 and 2016. In total, 18 tests were distributed in 2015's group and 17 in 2016's group. In summer term 2016, students were sent an anonymous questionnaire through Google after that. This guaranteed anonymity of respondents and they answered the questions from the questionnaire truthfully, because students knew results of the test.

Table 1: Answers of respondents for the questionnaire

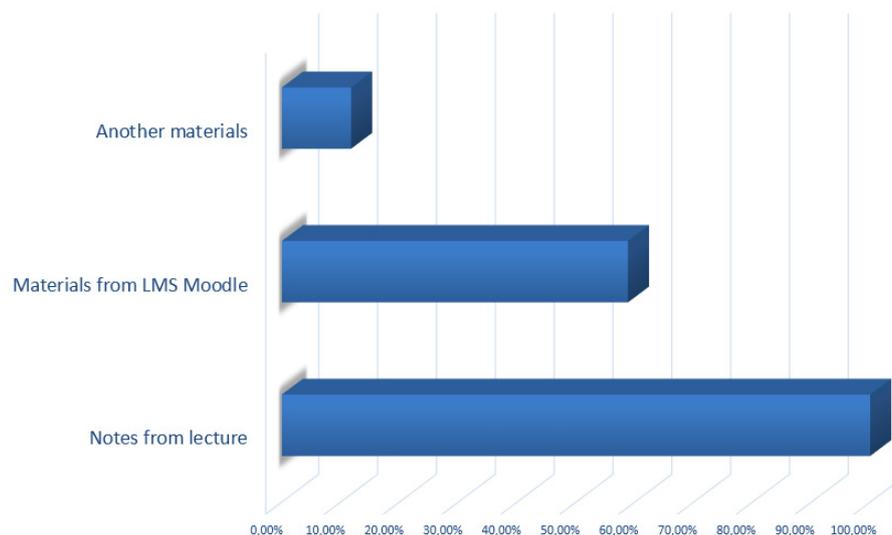|	Yes	No
Paper models were useful	88,24%	11,76%
Assembling of models was difficult	58,82%	41,18%
Did the model help you for better understanding of Monge projection?	82,35%	17,65%

Figure 6: The graph of What materials did you use to prepare for an exam?

For students, standard lectures are still important, because they very often used their notes for preparation for an exam. In our case, 100 % students used notes and same 59 % students used materials from LMS Moodle. Most of them didn't need to find and use another materials, only 12 % of students did it.

These days, in age of computer simulations, creation of paper models may seem as a "step back", but these models were very useful for our students. If we assembled these models in the lecture, students considered it very interesting and a climate of group was open and excellent. On the other hand, 59 % of students claimed

that assembling the models was difficult for them. Most students feel that the use of these models led to better understanding of the Monge projection.
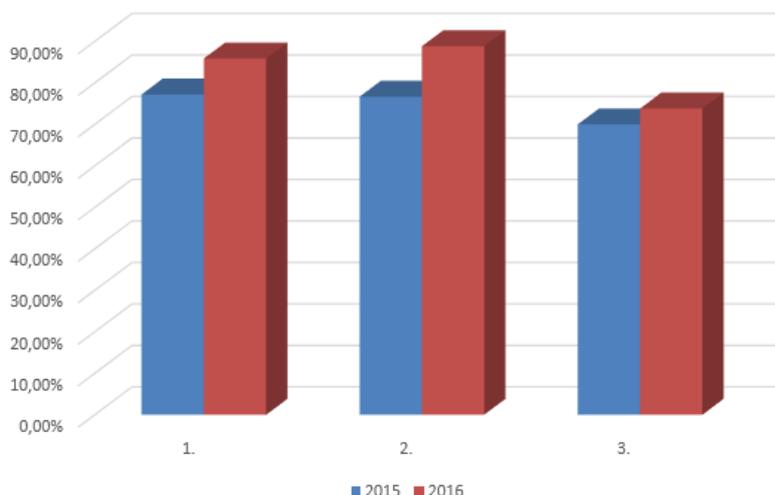

Figure 7: Average point achievement of students from Constructional Geometry 2015-2016

We set the test hypotheses that were tested on the acquired data using Wilcox test, because normality of data was rejected by Tests of Assumptions (Skewness Normality test and Omnibus Normality test). We used statistics application NCSS 8.0.

Hypothesis 1.
Ho: Students from summer term 2015 and 2016 will acquire equal number of points from Task 1.
H1: Students from summer term 2015 and 2016 will not acquire equal number of points from Task 1.

Table 2: Wilcoxon (W) Signed-Rank Test for Difference in Medians for Task 1

	Approximation Without Continuity Correction			Approximation Without Continuity Correction		
	Z-value	Prob. Level	Reject H0	Z-value	Prob. level	Reject H0
Median <>0	3,7412	0,000183	Yes	3,7167	0,000202	Yes
Median <0	3,7412	0,999908	No	3,7656	0,999917	No
Median >0	3,7412	0,000092	Yes	3,7167	0,000101	Yes

We reject zero hypothesis and accept alternative hypothesis. Students from 2016 were better than students from 2015 in task 1, (Table 2). Their average point achievement increased by 8.66% (see Figure 7).

Hypothesis 2.
Ho: Students from summer term 2015 and 2016 will acquire equal number of points from Task 2.
H1: Students from summer term 2015 and 2016 will not acquire equal number of points from Task 2.

Table 3: Wilcoxon (W) Signed-Rank Test for Difference in Medians for Task 2

	Approximation Without Continuity Correction			Approximation Without Continuity Correction		
	Z-value	Prob. Level	Reject H0	Z-value	Prob. level	Reject H0
Median <>0	3,7412	0,000204	Yes	3,6898	0,000224	Yes
Median <0	3,7412	0,999898	No	3,7383	0,999907	No
Median >0	3,7412	0,000102	Yes	3,6898	0,000112	Yes

We reject zero hypothesis and accept alternative hypothesis. Students from 2016 were better than students from 2015 in task 2, (Table 3). Their average point achievement increased by 12,15% (see Figure 7).

Hypothesis 3.

Ho: Students from summer term 2015 and 2016 will acquire equal number of points from Task 3.

H1: Students from summer term 2015 and 2016 will not acquire equal number of points from Task 3.

Table 4: Wilcoxon (W) Signed-Rank Test for Difference in Medians for Task 3

	Approximation Without Continuity Correction			Approximation Without Continuity Correction		
	Z-value	Prob. Level	Reject H0	Z-value	Prob. level	Reject H0
Median <>0	3,6346	0,000278	Yes	3,6109	0,000305	Yes
Median <0	3,6346	0,999861	No	3,6584	0,999873	No
Median >0	3,6346	0,000139	Yes	3,6109	0,000153	Yes

We reject zero hypothesis and accept alternative hypothesis. Students from 2016 were better than students from 2015 in task 3, (Table 4). Their average point achievement increased by 3,82 % (see Figure 7). This small increase is caused by a higher complexity of the task in comparison to previous ones.

6. Conclusion

Quality materials for self-study are an integral part of every teaching as well as quality usage of LMS Moodle. With possibility of usage of GeoGebra plugin in LMS Moodle, LMS obtains a new wave in process of self-study. The question is where the boundaries of the amount of generated materials are. We introduced the concept of materials for Monge projection here, especially on the materials of paper models and anaglyph models of the same tasks. For students, these materials are met with a warm welcome and are largely understood as refreshment of lecture. Respondents were students of course on Constructional Geometry. For testing, these students have a high inner motivation and this course is classified as very useful and necessary for their future work. After the pilot testing, we found out, that results of students were about 8 % better in comparison to previous year. It is also interesting, that these students did not meet the Monge projection in the previous study in high school. These results are motivation for us for our future work, because we would like to test the biggest group and would like to extend materials.

Acknowledgements

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