Using the Probabilistic Model Checker PRISM to Analyze Credit Card Use

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Abstract: Probabilistic model checking is a recent extension of traditional model-checking techniques for the integrated analysis of both *qualitative* and *quantitative* properties of systems which exhibit stochastic behaviour. In this paper we apply probabilistic model checking to study the effect of credit card companies on people's lives. We use the probabilistic model checking tool PRISM as the formal framework. This approach allows us to obtain performance measures on various policies. It allows us to obtain performance measures on various policies such as changes in the interest rate and its effect on the credit card loan entitlement of the card user, the effect of different repayment policies on the user's spending ability in the short and long run, the effect of different interest rates and different spending preferences on the loan in the short run, and the effect of different spending preferences and different repayment policies on the remaining balance in the long run.

From the study we investigate the level of loans and the amount of instalments after which the card holder goes through a cycle of interest repayment only. That is he/she can not use the card to withdraw money any more but has to make interest payments on the debt.

Keywords: credit card system, performability, probabilistic model checking, simulation.

1. Introduction

Probabilistic model checking is relatively new development which plans to bring automatic verification technology for real life systems which exhibit stochastic behaviour [1], [2]. The technique allows for analyzing of reliability, correctness and performance of such systems in an automatic way. Probabilistic Symbolic Model Checker (PRISM) is a probabilistic model checker which supports a range of probabilistic models and specification languages based on temporal logic, it allows accurate computation for a wide range of numerical properties and it performs a complete analysis which enables studies such as best and worst case scenarios. In the 21st century credit cards became one of the most popular methods of financing consumption. They are easy to use; they can be used in stores or on the net, and they can be used to withdraw money from banks or automated teller machines (ATMs). People in many countries around the world prefer using credit cards for safety reasons, instead of carrying cash in their pockets, and/or to utilize credit card product offers designed to meet their financial needs, since credit cards give them the opportunity to avail of more finance than they already have and pay later with "buy now, pay later" option.

In Egypt more than one million credit cards are in use now. The number of credit cards increased three times in Egypt since the year 2000, with spending rate more than one milliard dollar.[1] The monthly interest rate on the loans ranges from 1.4% to 2.5%. In the US paying by credit card gives more protection to the user than making payments by check or by debit card. [2] This creates incentives to use credit cards even more as a form of finance. In the first quarter of 2002 the total U.S. credit card debt was around $60 billion.[3] In 2004 an average American family had a credit card debt of about $4,000 and was paying around $800 a year on credit card interest.[4] In February 2006 in the UK total credit card debt was £56.4 billion.[5]

Credit card use has both advantages and disadvantages. As allured to above, the possibility of increased availability of financial means for spending and the security given by carrying credit cards instead of cash are the main advantages of credit card use. Main disadvantage of credit card use comes if the user increases his credit card purchases thereby approaching more and more his credit card limit. This in turn increases his

[1]http://arabic.cnn.com/2005/business/12/25/million_visa_card_egypt/index.html
[2] For instance, section 5, "Billing errors and overcharges", of the U.S. Fair Credit Billing Act gives a lot of safeguards to credit card users which do not apply for payments by check or by debit card.
[3] American Consumer Credit Counseling www.consumercredit.com
[4] www.investopedia.com
[5]Bank of England Figures.

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interest payments and reduces the quality of life of the user in the long run.[6] Consequently the borrower might suffer from bankruptcy and end up in a worsened condition than before starting to use the credit card. Credit card debt is about the most expensive way there is to finance consumer goods and services. In fact, with banks paying as low as 3.05% before tax (current base rate 4.5% less 1.45%)[7] annual interest rates on credit cards average 15.68%[8] in UK, around 11 percentage points above Bank of England's base rate, making it far too expensive for card users to carry any balance over the grace period.[9] In the US the average rate of interest on credit card debt is between 17-20%. In order to reduce debt more quickly and to pay less interest the user of a credit card should try to repay more than the minimum amount required each month. However, many instances borrowers are unable to repay their debts and credit card issuers are being forced to write off increasing levels of bad debt. According to the Bank of England unsecured consumer debt written off by banks increased from £2.9bn during the first nine months of 2004 to £3.7bn during the same period of 2005.In face of increasing debt problems credit card issuers as well as governments are looking for strategies to reduce loses incurred by banks and also, not least, to help over-indebted borrowers. Besides having to increase their bad debt provisions, banks often issue credit cards only if applicants can provide good credit ratings and history. In 2006 the UK government provided an amount of £45m to recruit 500 debt advisers to help increasing number of people with debts. This paper studies and analyses the effect of the credit card on the quality of the holder's life using the probabilistic model checker PRISM. The paper tries to analyze the effect of interest charges on the loans taken by the credit card user, the effect of different repayment policies of the user, and the possibilities of using the card without risking bankruptcy. The paper is organized as follow; next section presents a brief review of probabilistic model checking and an overview of PRISM; the tool used for analyzing the application. Section 3 describes how credit card system works, and presents relevant research in this field. Section 4 describes the model. Section 5 analyzes the results of the experiments. Conclusions are presented in Section 6.

2. Probabilistic model checking

In the last decade, computer scientists have made tremendous evolution in developing tools and techniques for verifying requirements of systems. One of the successful approaches is *model checking* [3], [4]. The essential aim of model checking is to verify finite-state systems algorithmically and formally thereby discovering whether a model of a system satisfies a given property. The specification is often written as temporal logic formulas. By checking design requirements, we search whether the system under study is realizable, whether the requirements are suitably modular and well structured. If incorrect requirements are implemented, incorrect system behavior might result requiring at least rework and maintenance which in turn would cause high expenses. In some cases implementing incorrect system behavior could cause huge catastrophe**s** such as loss of life and property.

In order to improve the quality of various aspects of requirements and design modelers apply automated tools to check the quality of the model. Figure 1 shows the model checking approach.

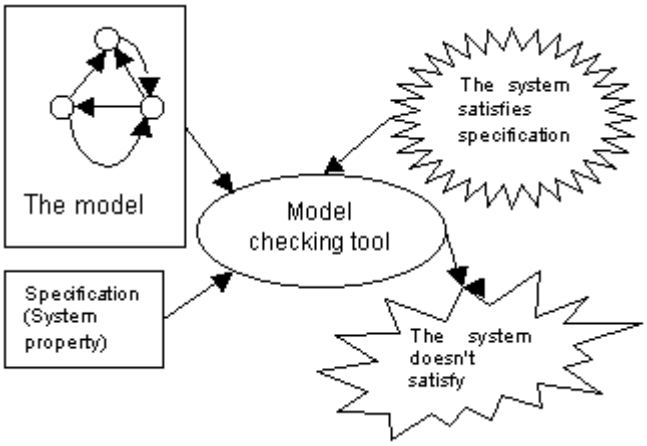

Figure 1: The model checking approach.

[6] See Ratha 1997.
[7] As of May 2006.
[8] As of February 2006
[9] The grace period is the time after the billing date that a credit card holder has to pay off the bill without paying finance charge. Bank cards usually have 25 days but a few have 30 and many have no grace period.

A *model-checking tool* accepts a *model* and a property (called *specification*) that the system is expected to satisfy. The tool then gives either the result with output yes if the given model satisfies a given specification or shows an error message otherwise. In the case of detecting an error the tool gives example as to why the model does not satisfy the specification. From this example the modeler can identify the source of the error in the model, correct the model, and run the model checking tool again. Thus, model checking tools ensure whether a model satisfies system properties sufficiently and help to increase confidence in the correctness of the model. There are many model checking tools such as Spin [5], NuSMV2 [6] and MARIA [7].

Probabilistic model checking is a relatively recent development which aims to deliver automatic verification technology for probabilistic systems. These systems are usually represented by discrete or continuous time Markov chains [8] or Markov decision processes [9]. In discrete time Markov chains (DTMCs) time as well as probabilities of transitions occurring are considered as being discrete. They are used for studying systems with simple probabilistic. Markov decision processes (MDPs) extend DTMCs by allowing a combination of nondeterminism and probability. They are suited to model multiple probabilistic processes that execute in parallel or to model systems with unknown behaviour. Continuous time Markov chains (CTMCs) model systems with continuous time, through the use of the negative exponential distributions, allowing accurate representation of the timing characteristics of e.g. component failures and job arrivals. This technique, as in the case of traditional model checking, involves building a model to represent a system, and then performing a systematic analysis on that model to check whether it meets its specification such as "in an electrical power station, the probability of shutdown occurring is at most 0.01%", and "what is the probability that the process will successfully complete within T time and without requiring repairs".

Real life systems are complex, and therefore there is a need for high-level specification techniques to automatically generate models. Now there are several tools available for probabilistic model checking, such as E ├ MC^2 [10], Rapture [11] and PRISM [12], [13], [14], [15]. These tools can enhance the design process through early error detection and prediction of failure and performance.

PRISM is a probabilistic model checking tool which has already been successfully applied to verify and analyse wide range of real-life systems [14], [16], and [17]. Parker et al worked on different PRISM applications[10]. PRISM can model three types of probabilistic models: DTMCs, MDPs and CTMCs. The PRISM tool has two files as input; the first one is the model file, which can be written in either the PRISM language or PEPA (Performance Evaluation Process Algebra) language [18]. The model file contains a description of the system. The second file is a specification file which contains a list of properties to be checked for conformity with the model.

The tool uses the Probabilistic Computation Tree Logic (PCTL) for properties of DTMCs and MDPs, and Continuous Stochastic Logic (CSL) for properties of CTMCs. After building the model PRISM performs numerical analysis to verify properties against it. The results state whether a property is in conformity with all reachable states of the model or the probability of the property conforming to the model. Rewards and costs are added to PRISM properties in order to describe how rewards should be accumulated for particular states of the model, and for particular transitions between states. PRISM structure for CTMC model looks as shown in Figure 2.

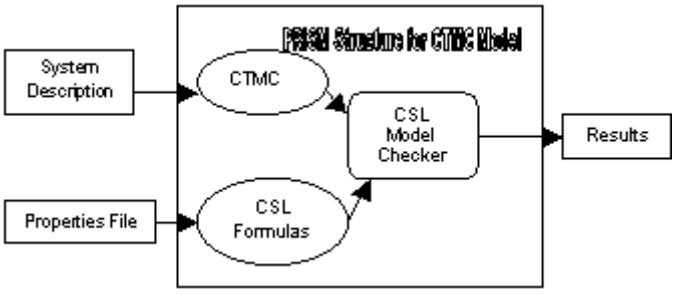

Figure 2: the structure of PRISM for CTMC model

The PRISM language consists of modules, which represent the components of the system. A module contains two parts: its variables and its guards. The variables are called local variables; they can be integers or booleans. The guard is a predicate over all the variables in the system. PRISM model can include global variables which are known to all modules. A module can transfer from its state to a new one through an

[10] see [17] for different PRISM applications

action. A module can make this action alone, i.e. independently, which is known as *asynchronous* transition, or the action is shared by two or more modules at the same time, which is known as *synchronous* transition. In CTMCs a time delay is incurred before a transition occurs, and a delay is exponentially distributed with rate *r*. The average time of transition is 1/*r*. There is more than one transition that can occur in a given state, the transition which is selected is the one that is enabled first. This is known as *the race policy*. The state of the whole model is determined by the states of all the modules together with the contents of the global variables.

Each module contains a set of commands. Each command for a particular predicate on the global state (guard) describes how the local state and global variable set can be updated, i.e. the transition that can occur if the guard is true. Property specification in PRISM for CTMCs is based on CSL probabilistic temporal logic. The principal operators calculate the probability of an event occurring, the long run probability of some condition being satisfied and the expected value of the model's costs or rewards. Experiments can run by PRISM tool. This allows having outcomes of one or more properties. It gives a way of automating multiple instances of model checking. PRISM also includes support for the specification and analysis of properties based on costs and rewards. This gives PRISM the ability to reason about expectations, such as “expected time” and “expected number of given items”. A single reward item can assign different rewards to different states, depending on the values of model variables in each one. The states that do not satisfy the guard of any reward item will not have any rewards. For the states (transition) that satisfy multiple guards, the reward assigned to the state (transition) is the sum of the rewards for all the corresponding reward items. Furthermore, PRISM supports the notion of experiments, which is a way of automating multiple instances of model checking. This allows the user to easily obtain the outcome of one or more properties as functions of model and property parameters, using the same assignments of model states. The most recent addition of PRISM is provided by a simulator. It is a tool that allows further analysis of probabilistic models. It calculates and reasons about execution paths through probabilistic models using Monte-Carlo methods and discrete event simulation.

3. Credit card

Let us study the effect of the use of a credit card on the quality of the user’s life in the long run.

Consider a person who has a fixed monthly income / salary equal to S. This person has a credit card which gives him the opportunity to increase his spending by H times, i.e. the person can spend amount equals HxS. The credit card issuing company pays for his spending. By the end of each month the company sends a statement with the total amount of money that the card holder spent using the card, and gives him a certain period of time for repayment. Let us assume that the period for repayment is one and half months. The card holder has the opportunity to repay all the money back or pay only part of it; in the latter case the company will charge interest rate on the rest of the money "rest balance" which has not been paid. The amount of money the card holder can spend on his card "available", is reduced each time he uses the card, and also by the amount of the interest rate added to the unpaid part, if there is any. If the card holder repays all the amount of money loaned by the card company, he will not suffer from increased debt caused by interest expenses. Also the more he repays the less the company can charge as interest. Many papers have discussed the credit card system from different research points. In [19] a credit card puzzle is presented for studying the behaviors of people using credit cards. In [20] a simple model of precautionary demand for money is presented, where a model gives an explanation for how the use of credit-cards can differ so widely across countries. In [21] a simple system dynamics model for credit card **use** is presented.

4. The model description

The PRISM model of the system consists of two modules, i.e. one module for the card holder and the other for the company. Figure 3 is the state chart diagram that describes the behaviour of the two modules.

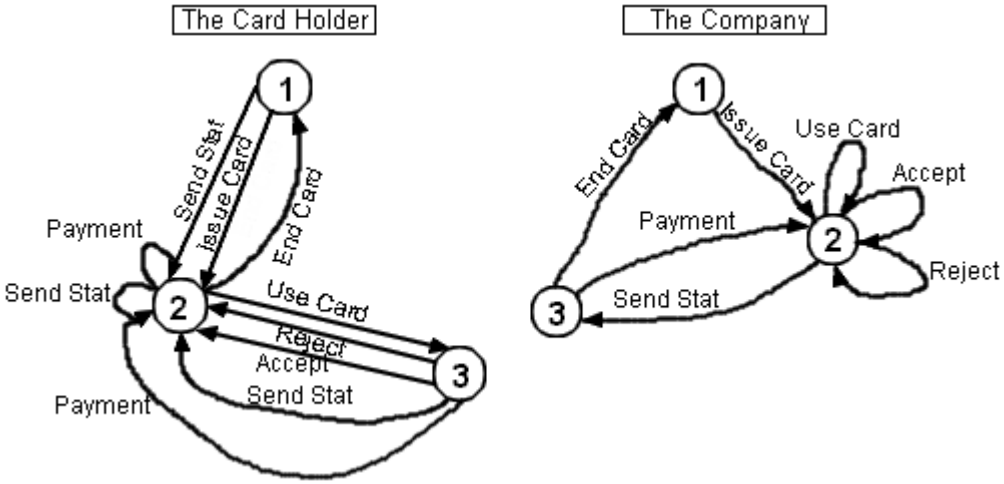

Figure 3: The state chart diagram of the behaviour of the two modules

Table 1 shows the activities shared between the two modules and their meanings.

Table 1: The activities and their meaning in the model

activity	Meaning
Issue card	The company issues a new card for the new card holder
Use card	The card holder uses the card in the machine
accept	The machine accept the order of using the card
reject	The machine rejects the order of using the card
End card	The card holder (the company) decides to end the card
Send statement	The company sends a statement with the due amount for the card holder
payment	The card holder pay back all or some of his due amount of money

There are seven variables in PRISM model; user, company, *new balance*, *pay balance*, *rest balance*, *new due* and *available*. The initial values for all of them are zero.

Let us follow the activities that the card holder involves with the card issuing company. When the person applies for a credit card his credit-worthiness is determined in order to find out whether he is eligible for a credit card. A credit card is issued (*issue card* activity), if the applicant meets the credit card eligibility criteria of the card issuing company. The card holder receives an amount, denoted as *available* value, equal to the "limit" which is H times the income (S) of the card holder. After issuing the card the card holder can use the card to purchase goods and services or use it to withdraw cash; this is done through the activity *use card*. The company, through automated teller machines can *accept* this activity or *reject* it. It accepts use of the card if the amount of money needed, X, is less than or equals to the available amount of money on the card at the time of use, otherwise the company will reject to process the activity *use card*. In case of acceptance the company evaluates the new *available* value by deducting X from the current amount of *available*. The value of *new balance* is increased by the value X. Every month the company sends a statement to the card holder with the *new due* amount of money that he owes to the credit card company. *New due* or *new balance* is calculated by adding both *rest balance* and the *interest due* on it, where *rest balance* is any amount of money owed by the card holder according to previous statements. When the card holder makes a payment (*pay balance)* the company calculates the new *rest balance* by subtracting *pay balance* from *new due* and the new value of *available* is the limit less *rest balance,* and the new values of both *new balance* and *new due* are zero. The company will accept to end the card use when the values of both *new balance* and *rest balance* are zero.

5. Analysis of the experiments and results

We use both PRISM simulator and the analytical method for running the experiments to analyze the topic under study. First we use the simulation technique to study the model for the short term by following the monthly payment statements. First we study the effect of different repayment polices and how they effect the spending ability of the card user. Consider that the card holder's salary equals 1500 units of money, and the credit card issuing company grants him a limit equaling to twice his salary. Interest rate equals to 0.01167. The card user decides to add 500 units of money each month to his salary by taking this money using the card. We study this case with different polices of repayments; repay a quarter, repay half and repay three quarters of his borrowing, respectively as shown in Table 2 and Figure 4.

Table 2: The effect of different polices of repayment on spending ability

Month	Spending ability with the salary	Spending ability with repaying a quarter	Spending ability with repaying half	Spending ability with repaying three quarters
1	1500	0	0	0
2	1500	1625	1750	1875
3	1500	1340	1623	1843
4	1500	1124	1559	1835
5	1500	959	1526	1833
6	1500	834	1510	1832
7	1500	740	1502	1832
8	1500	668	1498	1832
9	1500	614	1496	1832
10	1500	572	1495	1832
11	1500	541	1494	1832
12	1500	517	1494	1832
13	1500	499	1494	1832
14	1500		1494	1832
15	1500		1494	1832
16	1500		1494	1832
17	1500		1494	1832
18	1500		1494	1832
19	1500		1494	1832
20	1500		1494	1832

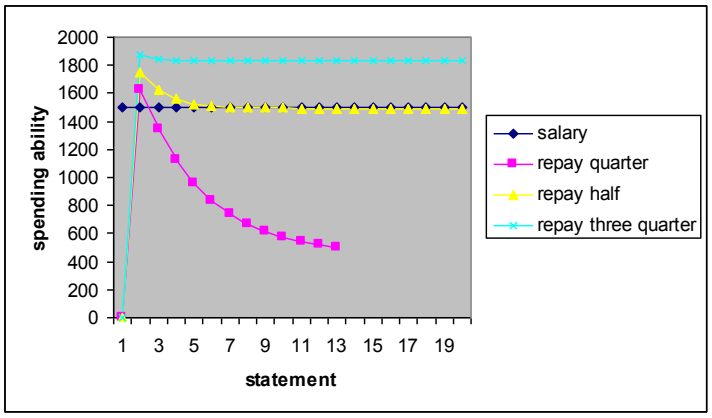

Figure 4: The effect of different polices of repayments on spending ability

Table 2 and Figure 4 show how much the card holder can spend each month according to these different repayment policies. We notice that if he decides to repay only a quarter of his loan, then the loan will accumulate and after one year (12 months) the company will block the use of the card (at least until he repays the loan). In the case when the card holder repays half of his loan we notice that after 8 months he starts to live with an amount of money less than his salary. If the card holder decides to repay three quarters of the loan he chooses the right policy, although he will spend less money than what he wishes but he will avail of more than his own salary every month (1832 units of money). Another case we study using the simulation technique is the effect of the change of the amount of money that the card holder takes every month and the interest rate that the company charges for delayed repayments by the card holder. We consider two cases; one where the card holder uses all the available amount of money and one where he uses three quarters of the available money. In both cases he repays three quarters of the loan. We study these two spending behaviors with two different interest rates, notably with 0.01167 and 0.025. Table 3 and Figure. 5 show the results.

Table 3: The effect of different spending money polices and different interest rates on the loans

Month	The loans when the user takes all the available & interest rate 0.025	The loans when the user takes all the available & interest rate 0.01167	The loans when the user takes three quarter the available &interest rate 0.025	The loans when the user takes three quarter the available & interest rate 0.01167
1	0	0	0	0
2	750	750	1500	1500
3	956	946	1538	1518
4	1013	998	1539	1518
5	1029	1011	1539	1518
6	1033	1014	1539	1518
7	1034	1015	1539	1518
8	1034	1015	1539	1518
9	1034	1015	1539	1518
10	1034	1015	1539	1518
11	1034	1015	1539	1518
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19	1034	1015	1539	1518
20	1034	1015	1539	1518

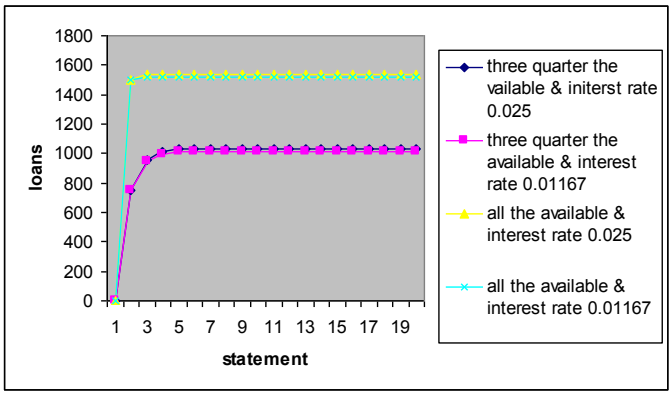

Figure 5: The effect of different spending money polices and different interest rates on the loans

As shown in Table 3 and Figure 5 for all different policies of repayment the card holder will end up in a deteriorated financial situation than if he does not use the card, due to an accumulation of the debt. Also, as the interest rate increases the debt will increase. Also we notice that after some period of time (depends on the spending behavior and the interest rate) the loan will stay at the same level due to this each month the card user goes through a cycle of repaying a fixed amount of the loan with interest charges due and as a result the amount that he can spend becomes limited.

Now we apply PRISM experiment for different values of repayment to study the system for the long run. Consider that each time the card holder uses the card he takes an amount of money equaling to M. Also consider that every time he makes the payment to the company he repays an amount equaling to (3×loan / K), where K is an integer value.

First, let us study the effect of different repayment policies on the accumulated loan. We fix both salary=100, H=2, interest rate=0.025% and let M= (3/4× available). Let K takes the values 4, 6, 8,…, and 30 (i.e. the card holder pays back three quarters of the loan, half of the loan, …, tenth of the loan). The expected rewards (loans) are calculated by running the experiment for "Steady state" properties, R=? [S], where S is used to reason about the behavior of the model in the long run. Table 4 and Figure 6 show the results approximated

to the nearest integer. From these we notice that when K=4, i.e. the card holder pays a significant proportion of this loan, his remaining debt is little. For K=6, 8 and so on it is straightforward, and the loan increases with smaller amounts of down payment and amortization. When the card holder pays back tenth of the loan (K=30) the accumulated loan for the long run becomes 18 units of money, i.e., nearly fifth of his salary.

Table 4: The long run effect of different repayment policies on the loan

K	4	6	8	10	12	14	16	18	20	22	24	26	28	30
Accumulated loan	1.0	2.0	4.0	5.0	6.0	8.0	9.0	10.0	12.0	13.0	14.0	16.0	17.0	18.0

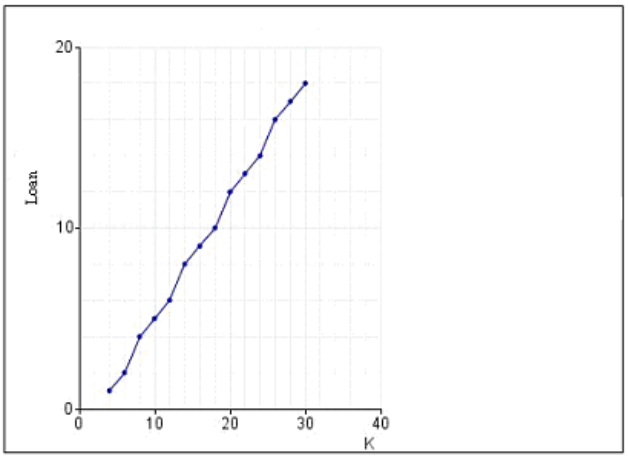

Figure 6: The long run effect of different repayment policies on the loan

Now we focus on a different spending preference and repayment policy of the card user and the impact of these on his quality of life. We fix the salary, H, and the interest rate as above. We change the values of both M and K. Let M takes the values 10, 60, 80 and 100, i.e., every time he uses the card, he spends a fixed amount of money equaling to M. Let K take the values as given in Table 5. The results are in Table 5 and Figure 7 approximated to the nearest integer.

Table 5: The long run effect of buying and payments policies on the spending ability

Spending ability according to different values of M and K	M=10	M=60	M=80	M=100
K=4	108	158	180	200
K=11	104	154	176	194
K=18	100	150	172	190
K=25	94	144	166	186
K=32	90	138	158	180
K=39	86	136	156	176
K=46	66	114	136	156
K=53	64	108	128	148
K=60	52	104	122	142

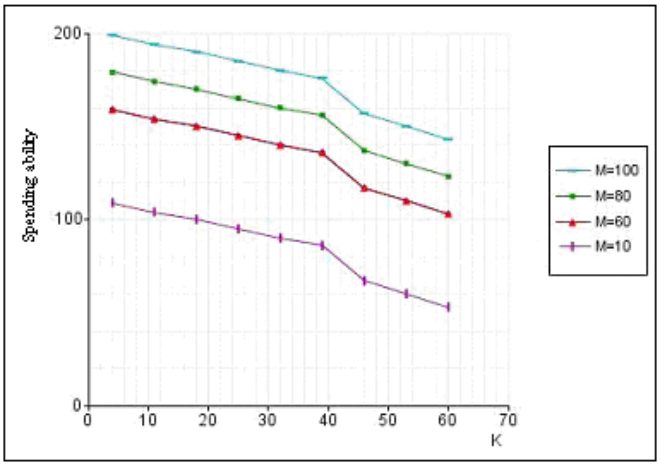

Figure 7: The long run effect of buying and payments policies on the spending ability

From Table 5 and Figure 7 we see that if the card holder borrows much and pays back much (three quarters of the loan) then his spending ability will be high. But if he pays back little (1/20 of the loan then his spending ability will be low.

6. Conclusion

In recent years, credit cards became the main source of purchasing used by consumers instead of cash payment. Credit card companies pay for the purchase, but the card user has to repay the money borrowed to the card company at a later time. In addition to the amount of the purchase, the card user also has to pay interest on the loan. Interest on a loan is calculated as a fixed fraction of the loan amount and is charged for the time period of the loan.

This paper, using the probabilistic model checker PRISM, evaluated credit card use under different conditions such as change in interest rates, different policies of borrowing/repayment and their effect on the credit card user's spending ability. It presents a model that studies the way interest payments affect the quality of the credit card user's life by reducing the amount of money he can spend on other purchases. The model can calculate the level of the money that the card holder can borrow according to his disposable income without going through a cycle of paying interest charges only.

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