Knowledge Maps and Mathematical Modelling

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Abstract: The aim of our paper is to explain a mathematical model as a special case of symbolic knowledge map. Each knowledge mapping is a visualization of knowledge for the purpose of eliciting, sharing and expanding. Tools of such visualization can be of various types. But in reality many types of so-called knowledge maps are only data flow or information flow diagrams. Our paper will define the most important features which every knowledge map must satisfy, for instance it must include chronological, hierarchical, associative, causal and evaluative relationships, it must improve the quality of knowledge etc. In our paper we will prove that a mathematical model satisfies all requirements to be called a knowledge map. Neither definition nor categorization and taxonomy of knowledge mapping are unified in the literature so the authors try to start with working on this field. Knowledge map is a visual interception of knowledge with the aim of its storage, sharing and development. Weak descriptive knowledge maps may be used for explaining the ideas and concepts connected with OR models, as well as for explaining the new knowledge gained with the models, in a well-structured form.

Strong descriptive knowledge maps can serve to describe real relations between the objects of the models or real elements in relation to their positioning. In this case the object placing does not describe only its physical position but also, for instance, its economical indexes. Like the normative OR models, the normative knowledge maps show the normative solution, or help to find the best, desirable or advisable solution. After suggestions of how to categorize knowledge maps (above) mathematical models of various types with all features and properties are presented as a knowledge map.

Keywords: knowledge map, knowledge map categorization, mathematical model, model construction, algorithm, model solution

1. Introduction

Mapping knowledge in its authentic substance has a template in geographical mapping, particularly military mapping, the roots of which stretch deep into the antique world. The first cartographers, who were already conscious of their limited knowledge, decorated their maps with various dragons and lions in the places, where exact data were absent. The maps documenting the ratio of knowledge in the face of ignorance arose this way, because knowledge in itself rises only on the basis of a successfully solved problem. Geographical maps were static in the principal points, but military maps included some dynamic features because of drawings or other graphic descriptions of the battle or the progress of its stages (pre-battle tactics, battle strategy, possible post-battle situations – many times in various scenarios). Much sooner, 30 000 years ago, the first cave paintings showing how to hunt a wild beast appeared. An unknown hunter codified his knowledge in the dynamic form for the purpose of sharing it with future generations. Knowledge mapping is the visualization of knowledge using a map, it means using non textual graphical form including a progress of problem solving for the purpose of its further reading, using, sharing and evolving.

2. Proposed knowledge maps typology

As mentioned above, the knowledge map is a visual representation of a successfully solved problem, including solving process (Stanford 2000). The solving process should contain at least four steps of the Simon's problem decomposition, i.e. intelligence activity, design activity, choice activity, and review activity (Simon 1960). Gordon (2002) also shows that knowledge maps may be referred to as maps of the way of acquiring knowledge. The knowledge maps are important as building knowledge tools as well as thinking tools (Rogers 2000). There are various definitions of the terms 'knowledge map' and 'knowledge mapping'. Stanford (2001) defines it as follows: "Knowledge mapping quite simply is any visualization of knowledge beyond textual for the purpose of eliciting, codifying, sharing, using and expanding knowledge." Graphic symbols play a key role in each knowledge map; their positions and spatial relationships are mostly expressed with the use of arcs or edges. The knowledge map must show a progression of ideas with relationships beyond their being just spatial. Knowledge maps include conceptual relationships, such as chronological, hierarchical, associative, causal, logical and evaluative (Stanford 2001). Each knowledge map, as a special type of reality model, for instance, a reality image, simplifies the visualisation of reality.

Similarly to the typology of models based on the model form, knowledge maps could be divided into two main groups:

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- Analogical maps and iconic maps, where the analogy between real objects and symbols, plus their spatial relationships and behaviour are crucial for the map understanding.
- Symbolic maps emphasizing the meaning of symbols, usually mathematical or verbal. This kind of maps generally doesn't insist on the symbol position. Elements of these maps are rather abstract (terms, expressions) and relations between them are expressed using mathematical formulas or verbal sentences or phrases.
- Another typology of knowledge maps is based on the character of judgment or solution of the (successfully) solved problem (Baron, 2004):
- Descriptive maps (weak and strong), describing and simulating the real situation as precisely as possible,
- Normative maps, relating to a typical standard or norm, to optimal solution, or to the best decision,

2.1 Weak descriptive knowledge maps

Weak descriptive maps describe the real situations using different kinds of symbols and arcs connecting them. Graph theory models are typical tools for building this kind of maps. Passing through this map helps the user to reach the problem understanding, increasing his/her level of knowledge of "how to" solve a problem. The mutual positions of objects (elements) are unimportant, only the symbols themselves and the quality of their relationships are relevant for map reading and problem solving. Conceptual maps (Figure 1) as typical weak descriptive maps are simple and practical knowledge representation tools that allow you to convey complex conceptual messages in a clear, understandable way. It helps to organize terms, concepts and other items mostly in a hierarchical way, where the most general concepts lie in the root of the tree, and as we move down the structure we replace them with the more specific ones. A conceptual map is a diagram showing the relationships between concepts. Concepts are connected with labelled arrows, in a downward-branching hierarchical structure. The relationship between concepts is articulated in linking phrases, e.g., "gives rise to", "results in", "is required by," or "contributes to" (http://en.wikipedia.org). Passing through this map means following the flows not only from general to specific but also from abstract to concrete.

Decision trees, flow charts representing algorithm progresses, and network diagrams for strategy implementation are only just a few representatives of this kind of maps in the field of Operations Research and Management Science.

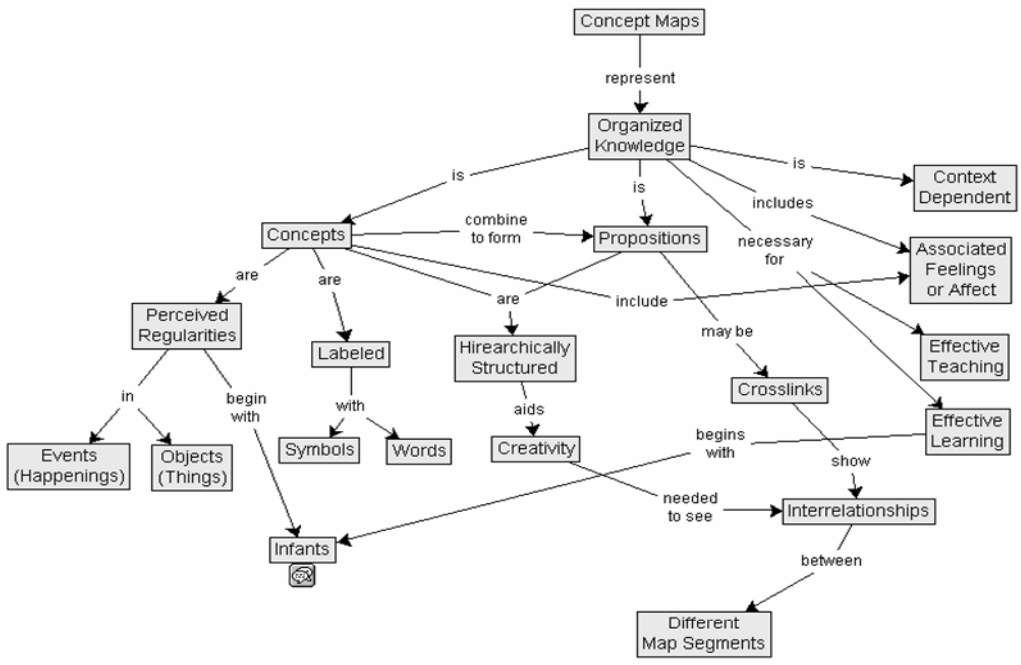

Figure 1: Concept Map (http://en.wikipedia.org)

2.2 Strong descriptive knowledge maps

Not only objects, symbols or texts are important for this kind of knowledge map. It includes more than text, such as symbols, legends and other visual objects. Thus to be a knowledge map the item must use spatial relationships to elicit, share and codify knowledge (Stanford 2001). Such knowledge map must show a progression of ideas with relationships beyond their being just spatial. Knowledge maps include conceptual relationships, such as chronological, hierarchical, associative, causal, logical and evaluative. Geographical maps are typical representatives of strong descriptive maps. Objects with properties and their spatial relations are mapped using isomorphic projection and a good quality quantitative (or sometimes qualitative) metric is needed for object distances measurement. As a quantitative expression of distance the closest distance between two nearest points of objects is considered. Not only the distance units can be used for such measurements - in special types of strong descriptive maps also costs, weights or points could be used.

Considering the distance ad its measurement, three types of spatial relationships are defined: proximity, adjacency, and containment (Figure 2).

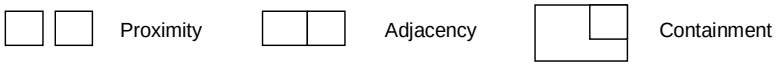

Figure 2: Spatial relationships types in strong descriptive maps

The rules for relationship types establishing:

- **Proximity**: Distance between objects *Ui* and *Uj* is non-zero, positive but small, i.e. $V(Ui,\ Uj) \in (0; M)$, where *M* is the upper limit of distance, where the objects still interact.
- **Adjacency**: Distance between objects *Ui* and *Uj* is equal to zero, both objects have a common interface, i.e. $V(Ui,\ Uj) = 0$.
- **Containment**: Distance between objects *Ui* and *Uj* is negative. i.e. ($V(Ui,\ Uj) < 0$). Objects can but need not have a common interface. According to the existence or non-existence of a common interface we define either partial or full containment.

2.3 Normative knowledge maps

In this case, the aim of the knowledge map is to introduce the approach of how to reach the target (solution), or of how to reach the comparative norm. Strategy maps cover the major part of this knowledge map type. Strategy maps are a way of providing a *macro* view of an organization's strategy, and provide it with a language in which they can describe their strategy, prior to constructing metrics to evaluate the performance against their strategy [en.wikpedia.org]. Strategy knowledge map is a technique of creation and use of graphical interpretation of situation in an organization or any other system. A Strategy Map is a diagram that describes how an organization creates a value by connecting strategic objectives. It describes organizations trends, main streams of effort (Mission and Vision statements) and targeting (way of reaching the norm). A good example of normativity of strategic maps can be shown by the example of the so-called Vee map (Figure 3). As its name suggests, the Vee map is of the letter "V" shape. It is a way of exploring the tension between the theory and the method and using this to gain and retain knowledge. The Vee map follows two axes extending down from the top to form a point at which they join at the bottom. The theory or concepts follow one axis and the methods or how-to follow the other. The problem statements or questions for examination are fed or funnelled down the centre between the two axes and eventually the assessment against each will bring the examiners to their conclusions or solutions (Stanford 2001).

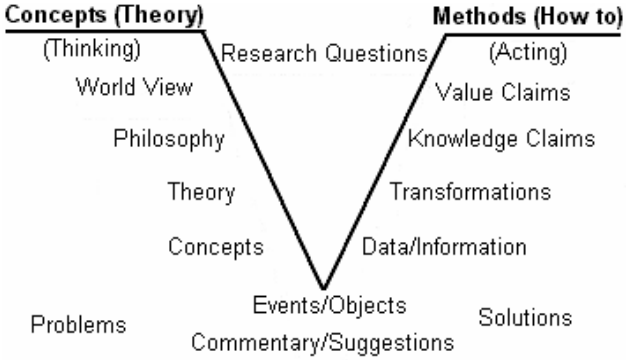

Figure 3: Basic vee map as a representative of normative knowledge maps (Stanford 2001)

3. Knowledge maps and mathematical models

3.1 When and why use mathematical models

Answer to this question can be found using a conceptual map describing the relation between solved problems, existing models and the process of application. This map can be characterized as a weak descriptive map showing the main steps of process solving. Brinkmann (2005) shows this type of knowledge maps as a tool to build structures in mathematics. Let's use the Leontief Input – Output model as a typical example of knowledge maps. The first map (Figure 4) represents the selection of the proper model type. This map is a normative map, because it shows which model has to be used for solving of different problems. The second one (Figure 5) shows the data that are necessary for the application of this type of model and relations between the model features, so it is a weak descriptive knowledge map.

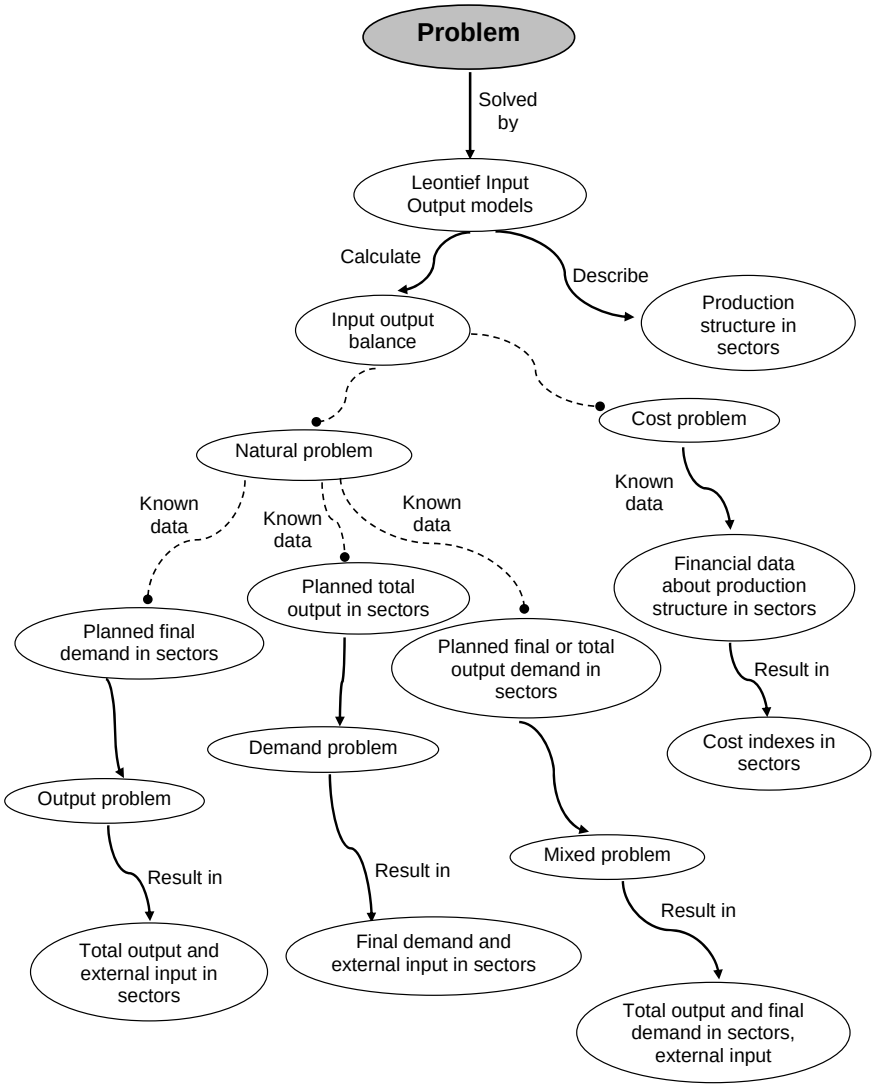

Figure 4: Normative knowledge map for the best model selection

3.2 Simulation models

What kind of knowledge map is a standard simulation model? Usually this kind of models is described (drawn) as a flow chart (Figure 6) – which shows a weak descriptive map. Only object symbols and quality of relations between them are important for the model simulation. The object placement is important only for objects consequence description and it is impossible to set some metric for this placement.

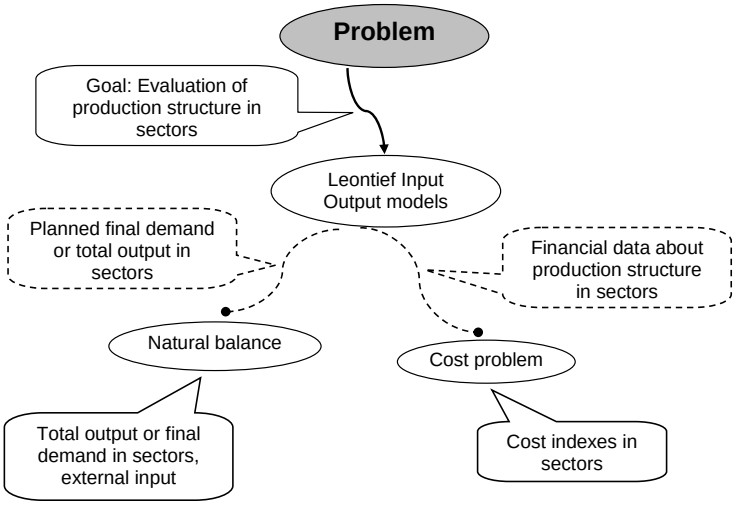

Figure 5: Weak descriptive knowledge map of the leontief models

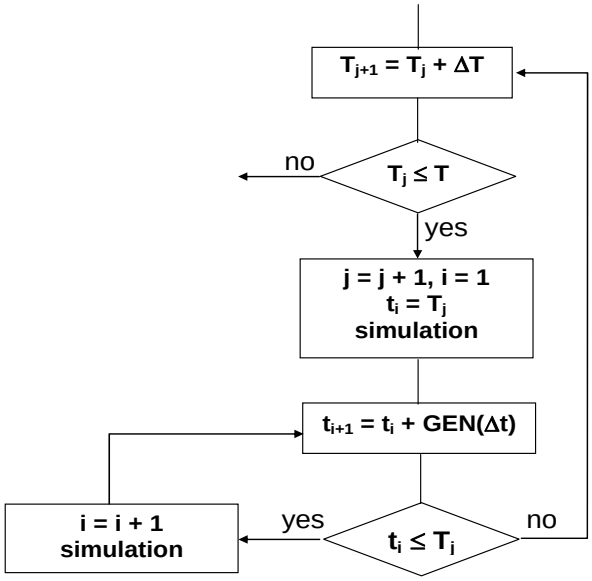

Figure 6: Example of a simulation model flow chart (weak descriptive map)

3.3 Decision tree

Decision tree is a graphical form of a decision model and it can be explained as a weak descriptive knowledge map describing decision situations, possible decision alternatives, and states of nature and sequence of these elements (Figure 7).

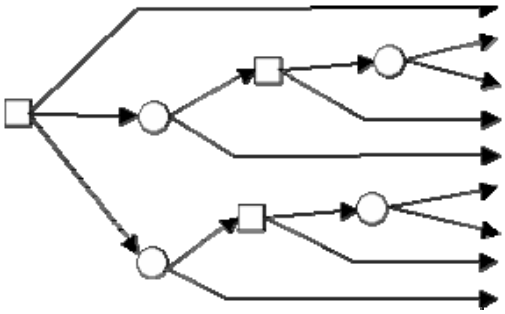

Figure 7: Decision tree – description of successive decisions

When we add rules for the best alternative selection we obtain a normative knowledge map leading to the normative decision (Figure 8).

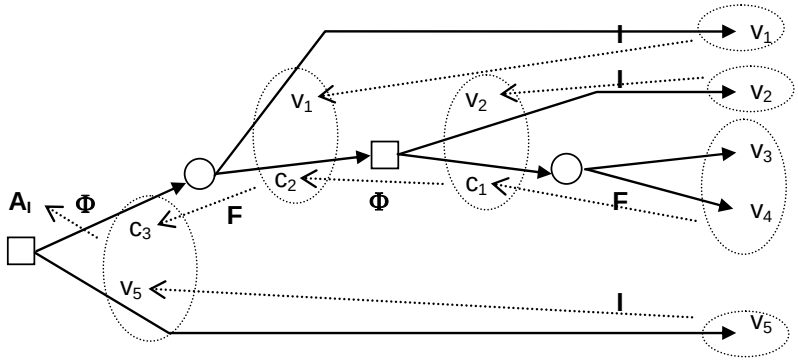

Figure 8: Decision tree with the best decision selection

3.4 Strategic knowledge mapping in project management

Conventional project management model is, from the mathematical point of view, a graph theory model or network model. Usually, the project tasks are drawn with the use of the nodes in this chart and the relationships between the nodes are expressed using arcs ("Activity on Node graphs). As mentioned before, the network model (sometimes called a PERT Chart) is a weak descriptive knowledge map. More sophisticated methods can be used for expressing the knowledge flow and transformation during the project tasks progress. One of them is a WBS chart (Figure 9), which describes the project tasks hierarchy without time information. This chart is also a weak descriptive map.

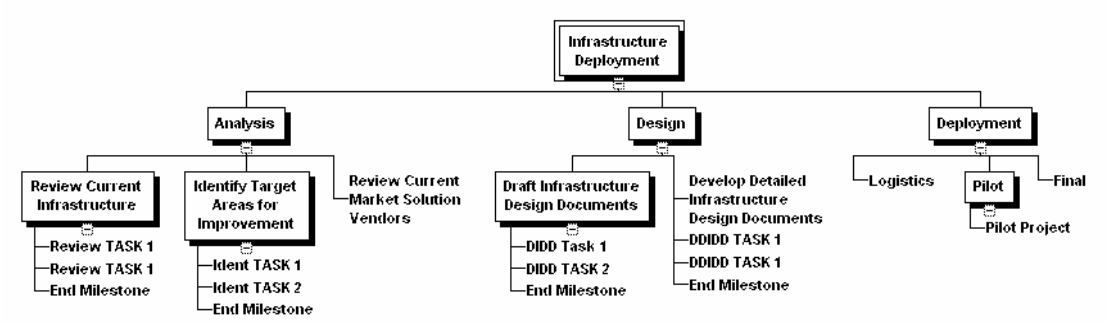

Figure 9: WBS chart (weak descriptive map)

The Gantt chart (Figure 10) places every task on a specific date and displays the tasks dependencies exactly on the time scale, and is therefore a strong descriptive map. Usage of timescale moves this kind of knowledge map one level higher – into the group of strong descriptive maps. Tools for the tracking project progress and comparison of the task finish dates transform this mapping into a normative knowledge map.

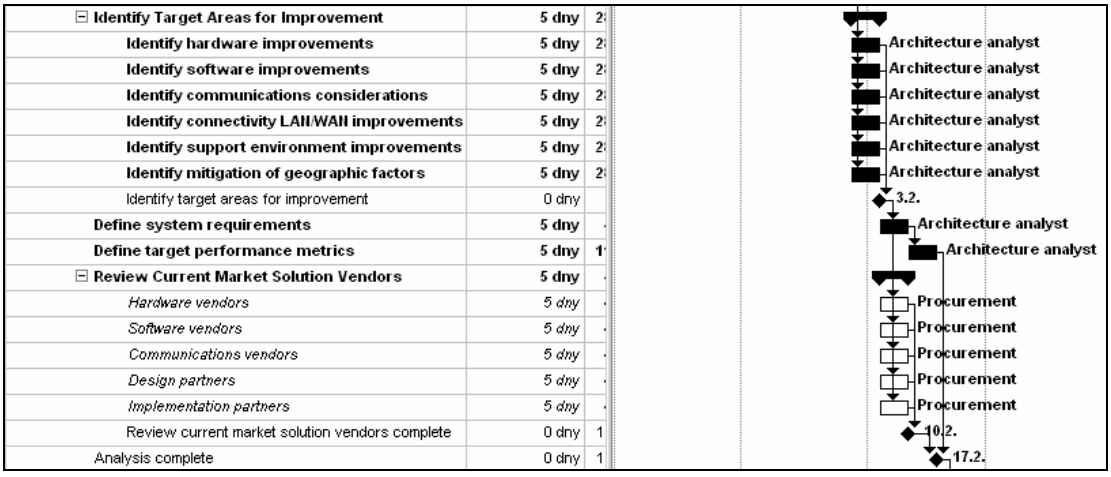

Figure 10: Gantt chart (strong descriptive map)

4. Conclusion

This paper suggests a new type of knowledge map classification based on OR models features. This approach arises from the idea that knowledge and application of the Operational Research models can be read as a graphical representation using different types of knowledge maps. Weak descriptive knowledge maps may be used for explaining the ideas and concepts connected with OR models, as well as for explaining the new knowledge gained with the models, in a well-structured form. Strong descriptive knowledge maps can serve to describe real relations between the objects of the models or real elements in relation to their positioning. In this case the object placing does not describe only its physical position but also, for instance, its economical indexes. Like the normative OR models, the normative knowledge maps show the normative solution, or help to find the best, desirable or advisable solution.

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